A COMPUTER PROGRAM FOR INTEGER SOLUTIONS TO LINEAR PROGRAMMING PROBLEMS

by

John Chaney Arick

United States
Naval Postgraduate School

THESIS

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John Chaney Arick
Captain, United States Marine Corps
B.S., U. S. Naval Academy, 1962

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ABSTRACT

An algorithm for the solution of integer linear programming problems is presented and programmed in Fortran IV for use on digital computers. The program incorporates an optional feature which provides all existing alternative optimal solutions. Solutions, computation times, and iteration requirements for each of thirteen test problems are summarized and discussed.

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I. INTRODUCTION

An algorithm presented by Dr. Harold Greenberg of the Naval Postgraduate School [1] provides an iterative procedure for the solution of the integer linear programming program: find

$X = (x_1, x_2, \ldots, x_n)$ which

minimizes

$$C\,X \tag{1}$$

subject to

$$AX = B$$

where

$$0 \leq x_j \leq b_j,\ x_j \text{ integer},\ j=1, \ldots, n.$$

C is an n-vector, B is an m-vector, A is m x n, and the components of C, B, and A, as well as the b_j are integer.

A bounded variable linear programming algorithm [2] is first utilized to transform (1) into its equivalent problem (i.e., a continuous solution to (1)):

minimize

$$d + \sum_{j \in H} c_j x_j$$

subject to

$$x_G + \sum_{j \in H} \alpha_j x_j = \alpha_o \tag{2}$$

where

$$0 \leq x_j \leq b_j,\ x_j \text{ integer},\ j=1, \ldots, n.$$[1]

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[1] The first five subroutines of the program accomplish this portion of the solution and were provided by Dr. Greenberg.

G is the set of indices of basic variables; H is the set of indices of non-basic variables; X_G, α_j, and α_o are vectors, with X_G the vector of non-basic variables.

If α_o is not all integer, the equivalent Knapsack Problem [3] is then formed:

minimize

$$\sum_{j \in H} c_j x_j$$

subject to (3)

$$\sum_{j \in H} \beta_j x_j = \beta_o \pmod 1$$

where $0 \leq x_j \leq b_j$, x_j integer.

The β's are the fractional parts of the α's from (2) above, and as such are also vectors.

The Knapsack problem is subsequently solved [1] using a dynamic programming enumeration which guarantees that all existing integer solutions can be found. Should there exist no feasible integer solution, the iterative process ceases after all possible column vectors of fractional parts have failed to produce a feasible solution.

II. THE ALGORITHM[2]

1. Let the indices, H, be 1, 2, ..., m. Form the following tableau (subsequently referred to as a section):

1, 2, . . . , m
C_1 C_2 $\quad$ C_m
β_1 β_2 $\quad$ β_m
$x_j=0$

and go to 2.

2. (a) Given the (new) section, test for $\beta_j = \beta_o$ for all $j \in H$. If, for one or more j, $\beta_j = \beta_o$, select the minimum C_j and call this C_r. If $C_r < C_r^*$, where C_r^* was the C_j which produced the last feasible solution, set $C^* = C_r$ and go to 4.

(b) If, for all unmarked j in the current section, $\beta_j \neq \beta_o$, find $C_r = \min C_j$ for all unmarked columns in all sections. If $C_r = C_r^*$, the current solution corresponding to C_r^* is the optimal solution to (1). If $C_r < C_r^*$, mark the column and go to 3.

3. (a) Form a new section in the following manner: compute $C_j' = C_r + C_j$, where the C_j are the values from the first section. If $C_j' \geq C_j^*$ do not add a corresponding column to the new section. Otherwise, continue by computing $\beta_j' = (\beta_r + \beta_j) \bmod 1$ for all $j \in H$ (the β_j are the values taken from section 1). For the $x_j = b_j$, $j \neq r$, and $x_r + 1 = b_r$ for $j = r$, for the section containing the newly marked column, do not add

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[2]The majority of procedures presented here are taken from the original algorithm [1].

the corresponding column to the new section. For columns which duplicate those already formed and for which the subsequent use of either column would result in identical values for the x_j, do not add a column.

(b) Underneath the new section, write the x_j values from the section containing the newly marked column. Increase x_r by one for the new section and go to 2.

4. Take as a trial solution the values of the variables found below the section where $\beta_r = \beta_0$ with x_r increased by one. If the constraints in (2) are satisfied, replace the current feasible solution corresponding to the old C_r^* with the new solution and go to 2 (b). If they are not, replace the C_r^* with the previous value and go to 2 (b).

Example:[3]

minimize

$$5x_1 + 7x_2 + 10x_3 + 3x_4 + x_5$$

subject to

$$\begin{aligned} x_1 - 3x_2 + 5x_3 + x_4 - 4x_5 &\geq 2 \\ -2x_1 + 6x_2 - 3x_3 - 2x_4 + 2x_5 &\geq 0 \\ - x_2 + 2x_3 - x_4 - x_5 &\geq 1 \end{aligned}$$

$0 \leq x_j \leq 1$, x_j integer, j=1, ..., 5.

The surplus variables are added ($x_j \geq 0$, j=6,7,8) and the problem is solved for its continuous solution which yields the equivalent problem:

minimize

$$9 + 93/9\, x_1 + 156/9\, x_4 + 42/9\, x_5 + 24/9\, x_7 + 81/9\, x_8$$

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[3]This problem originally appeared as an example problem in [1] and was also used as test problem #10 for the program presented here.

subject to

$$-7/9\ x_1 \qquad -28/9\ x_4 + 13/9\ x_5 + x_6 + 1/9\ x_7 - 21/9\ x_8 = 3/9$$

$$-2/9\ x_1 + x_3 - 8/9\ x_4 - 4/9\ x_5 - 1/9\ x_7 - 6/9\ x_8 = 6/9$$

$$-4/9\ x_1 + x_2 - 7/9\ x_4 + 1/9\ x_5 - 2/9\ x_7 - 3/9\ x_8 = 3/9$$

$0 \leq x_j \leq 1$; j=1, ..., 5; $x_j \leq 0$, j=6,7,8; x_j integer.

Section (1) is formed as follows:

1	4	5	7	8
93	156	42	24	81
2	8	4	1	6
7	1	5	8	3
5	2	1	7	6
*	*	*	*	*

$x_j=0$ (1)

with β_o = (3, 6, 3)

Since min C_j = 24, C_r = 24 and r = 7. Mark column 7 in section (1) and form the new section:

1	4	5	7	8
117	180	66	48	105
3	0	5	2	7
6	0	4	7	2
3	0	8	5	4
*	*	*	*	*

$x_\eta=1$ (2)

Note that in column 1 of section (2) $\beta_1 = \beta_o$. This means that an integer solution is obtained by substituting the non-basic variable values found below section 2 (with x_r, i.e., x_1, increased by one) into the equivalent problem objective function and constraints.

This yields the following solution:

$$z^* = 22,\ x_6 = 1,\ x_3 = 1,\ x_2 = 1,\ x_1 = 1,\ x_2 = 1,$$

which satisfies the feasibility requirements. Set $C^* = C_1$, i.e., $C^* = 117$ (C^* can initially be considered ∞). At this point, mark all columns for which $C_j \geq C^*$ (but not the C^* column itself) and select the next min C_j = 42. Mark column 5 of section (1) and form the new section:

1	4	8
135	198	147
6	3	1
3	6	8
6	3	7
*	*	*

$x_5=1$

Note that columns 5 and 7 have not been formed. The former because x_5 is now at its upper bound and can no longer be increased, the latter because it would duplicate column 5 of section (2) (the use of either will result in identical non-basic variable values). Also since $C^* = 117$, this entire section can be eliminated; use of any of its columns will result in a higher objective function value.

Seclect the next min C_j = 48, mark column 7 of section (2) and form the new section:

5	7
90	72
6	3
3	6
6	3
*	

$x_7=2$ (3)

Since $C_j > C^*$, $j = 1, 4, 8$, the corresponding columns have been omitted. Since column 7 has $\beta_7 = \beta_0$, for the integer solution:

$$z^* = 17,\ x_6 = 0,\ x_3 = 1,\ x_2 = 1,\ x_7 = 3.$$

which satisfies the feasibility requirements. $C^* = 72$ now replaces $C^* = 117$ and all previously formed columns for which $C_j > C^*$ are marked.

Select the next min $C_j = 66$, mark column 5 of section (2), and form the new section:

1	4	8
159	222	147
7	4	2
2	5	7
4	1	2
*	*	*

$x_7=1$ $x_5=1$

But $C_j > C^*$ for all columns of this section, so delete the entire section (column 5 had been omitted since x_5 is at its upper bound; column 7 had been omitted since it was a duplicate meeting the requirements for the omission of duplicates mentioned above). Select min $C_j = 72$ and note that $C_r = C^*$ which means the solution corresponding to $C^* = 72$ is optimal.

The algorithm as originally presented in [1] makes no provision for the recognition of an integer L.P. problem which, although it has an optimal feasible continuous solution, has no feasible integer solution. This means that such a problem, when operated on by a computer program utilizing the algorithm, will run indefinitely, or until space and/or time limitations are reached. This of course can be costly in terms of

computer time, so a method is presented here, and incorporated in the program, for recognizing such infeasibility conditions.[4]

Let

Δ = the common denominator of the coefficients of the equivalent problem,

g = (a, b) be the greatest common denominator of a and b,

β_j^T = $(a_{1j}, a_{2j}, \ldots, a_{mj})$, $j = 1, \ldots, n$, be the j^{th} vector of fractional parts of the first section,

$g_{ij} \equiv (a_{ij}, \Delta)$,

$g_j \equiv \min_{1 \le i \le m} g_{ij}$ $j = 1, \ldots, n$,

$p_j \equiv \Delta / g_j$ for all j, and

C_j = cost of $\vec{a}_j$.

Since the period, p_j, is actually the number of times column j can be added to any other column before the column j assumes its original values (mod Δ), it follows that

$$\vec{a}_j + p_j \vec{a}_j = \vec{a}_j \pmod{\Delta} \quad , \text{ or}$$

$$p_j \vec{a}_j = \vec{0} \pmod{\Delta}.$$

Therefore, the maximum number of distinct column vectors, M , which can be formed by the algorithm will be

$$M = \prod_{j=1}^{n} p_j \quad ;$$

this is because each column will assume p_j different intermediate values

when operated on by any other column, either consecutively or in combination with other columns, prior to reassuming its original values.

Note now that any vector, $\vec{K}$, generated by the algorithm can be written in the form

$$\vec{K} = \vec{a}_j + \sum_{\ell=1}^{n} \gamma_\ell \vec{a}_\ell \pmod{\Delta}$$

where $0 \leq \gamma_\ell \leq p_\ell$ for all ℓ. γ_ℓ is necessarily integer and determined by the number of times column ℓ was utilized in the formation of $\vec{K}$. Note also that the individual $\gamma_\ell a_\ell$ terms are actually the intermediate columns which were formed prior to $\vec{K}$ and which played a role in the formation of $\vec{K}$.

Since there is a definite upper bound on the number of distinct column vectors, a bound may now be placed on the costs which will be generated. The cost associated with any vector generated by the algorithm will be

$$C_j + \sum_{\ell=1}^{n} \gamma_\ell C_\ell \quad .$$

The last distinct column vector to be formed prior to repetition will be

$$\sum_{j=1}^{n} (p_j-1)\, \vec{a}_j \quad ,$$

and the cost associated with this is

$$\sum_{j=1}^{n} (p_j-1)\, C_j \quad .$$

By previous development, if we add any vector to the current one, say $j=\sigma$,

$$\sum_{j=1}^{n} (p_j-1)\, \vec{a}_j + \vec{a}_\sigma = \sum_{\substack{j=1 \\ j\neq\sigma}}^{n} (p_j-1)\, a_j \pmod{\Delta} \quad ,$$

since

$$p_{\sigma}\vec{a}_{\sigma} = \vec{0} \pmod{\Delta}.$$

Hence an upper bound (possibly not the least upper bound) on any cost, prior to all vectors being duplicated, will be

$$C' = \sum_{j=1}^{n} (p_j - 1)\, t_j \text{ , where } t_j = \min_j(C_j, 0) \text{ .}^5$$

If, at any point in the search for feasible solutions, C' is reached and no feasible solution is found prior to the generation of the next section, it can be assumed (and is by the program) that no feasible integer solution exists. This follows since it has been shown that all columns formed subsequently will duplicate prior columns which have already failed to produce a feasible solution.

Note that, given a large enough common denominator, the value of C' can become extremely high. However, the time and space requirements become prohibitively high at the same time, and efficient utilization of the program appears very unlikely for such problems. (This is discussed in detail in the section on Computational Results.)

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[5] The definition of t_j allows a loose upper bound to be placed on costs for problems for which the continuous solution has one or more non-basic variables at their upper bounds (i.e., $C_j < 0$).

III. PROGRAMMING CONSIDERATIONS

Because all integer solutions to the congruence in (3) are systematically produced in order of increasing objective function value while searching for a solution, the original algorithm [1] necessarily would require considerable core storage in order to accommodate each intermediate section. Accordingly, several changes are introduced, the first of which modifies the procedure for identifying feasible solutions.

Rather than limiting a check for feasible solutions to the min C_j column at each iteration, the new section is checked at the time of its formation for possible integer solutions. This allows the peremptory discarding of all columns, formed subsequent to the computation of a possible solution, for which the C_j are greater than the one which produced the current feasible solution (i.e., if the column were retained and subsequently used to produce a feasible solution, the solution could not be optimal since the resulting objective function value would necessarily be greater than the current one; $d + C_r^*$). The utility of this particular modification depends entirely on how early in the iterative process a possible solution ("base" C_r^*) is found.

The algorithm provides for the elimination of duplicate columns only if the use of either would result in identical non-basic variable values. The program, however, allows the elimination of all duplicate columns in an initial search for an upper/lower bound (depending on minimization/maximization) on the optimal solution to the problem. The program then reruns the problem using the procedure originally specified by the algorithm in a search for the optimal solution. The bound placed on the optimal solution by the initial run of the program may be the optimal

solution itself; however, as will be explained, there is no guarantee that this will, in fact, be the case.

By definition, a duplicate column contains duplicate fractional parts as well as C_j values. Hence, it can be shown that the objective function values will always be identical when duplicate columns are used to produce integer solutions; but the basic variable values will be guaranteed to be identical only when their corresponding non-basic variable values are equal. Therefore, when a duplicate column is allowed to remain, based upon the fact that when selected it will produce non-basic variable values different from those of the original column, it will subsequently produce a section identical to the one produced by its "sister" column but with a different combination of non-basic variable values. When a column from one of the identical sections satisfies the $\beta_j = \beta_o$ requirement for an integer solution, the different non-basic variable values can produce a feasible solution in one case and a non-feasible solution in the other. Hence, if one column were to be peremptorily discarded, a feasible solution may be overlooked, and although the subsequent formation of duplicate columns may cause this solution to "reappear", it will continue to be lost due to its duplication. Test problem #3 demonstrates how this "course" duplicate column elimination method will run out of time and/or space prior to reaching an optimal solution. This is to be expected in a number of problems since an entire section necessarily takes longer to become totally marked, and hence its space does not become available for reuse until much later in the iterations, if at all. (This will be discussed in more detail later in this section). The "course" method, therefore, provides at the very least a bound on the optimal solution, although in all of the test problems

included here, these bounds were, in fact, optimal solutions. The following example should make clear, however, why this will not always be the case.

Assume that the following equivalent problem has been computed:

$$\text{minimize} \quad \frac{212}{8} + \frac{5}{8}X_2 + \frac{10}{8}X_3$$

$$\text{subject to} \quad X_1 + \frac{3}{8}X_2 - \frac{2}{8}X_3 = \frac{4}{8}$$

$$\frac{5}{8}X_2 - \frac{6}{8}X_3 + X_4 = \frac{12}{8}$$

$$X_j \geq 0 \quad j = 1, 2, 3 \; ; \; 0 \leq X_4 \leq 2$$

Now form sections (1) and (2) according to the algorithm, noting that

$$\beta_0^T = (4, 4).$$

2	3	
5	10	
3	6	(1)
5	2	
*	*	

$X_j = 0$

2	3	
10	15	
6	1	(2)
2	7	
*		

$X_2 = 1$

Now form two new sections (a) and (b) first using Column 3, section (1) then Column 3, section (2).

2	3	
15	20	
1	4	(a)
7	4	

$X_3=1$

2	3	
15	20	
1	4	(b)
7	4	

$X_2=2$

Note that integer solutions may be found in columns 3 of (a) and 3 of (b). For the solution from (a), substitute $X_3 = 2$ into the equivalent constraint equations and get $X_1 = 1$ and $X_4 = 3$, an <u>infeasible</u> solution. For the solution from (b), substitute $X_2 = 2$ and $X_3 = 1$ into the equivalent constraint equations and get $X_1 = 0$ and $X_4 = 1$, a <u>feasible</u> solution.

Here, we see that if column 3 of (b) had been marked as being a duplicate, while column 3 of (a) had been retained, the solution would have been judged infeasible and all other solutions eliminated. This is, in fact, what happened in test problem number 9. It was not until the more strict "fine" duplicate column elimination criterial of the original algorithm were imposed that the "feasible" column was retained (this based upon the fact that, when chosen, it would produce different non-basic variables from its "sister" column) .

For the same reason that a feasible solution could have been bypassed in the example, one could also have been overlooked which would yield a lower objective function value than the current solution. For example,

consider the case where, prior to the formation of sections (a) and (b) above, a possible solution had been recorded which utilized the following column from a previously formed section:

$$(C_j \beta_j) = (28 \quad 4 \quad 4) .$$

It can be seen that the value of C_j here would yield a higher objective function value than either of the two columns previously discussed. The optimal solution would then be bypassed by virtue of the course duplicate column elimination criteria. However, should the program run out of time and/or space on the "fine" run prior to reaching the sections containing the columns above, we would at least have an upper bound on the minimum objective function value.

The two methods for duplicate column elimination are also made use of when the program is required to search for alternative optimal solutions. For the same reason that no optimal solution may be found on the first run, alternative solutions may also be bypassed. Hence, when the option to search for additional solutions is exercised, the program once again searches under both modes of duplicate column elimination. Time and space limitations may be binding here also, but given enough of both, the program will provide all alternative optimal solutions when required. In problems where two or more optimal solutions are found in the same section, all are recorded (whether or not alternative optima have been requested by the user) since no additional iterations will be required.

The principle space saving technique utilized by the program is one which makes section space available for newly formed sections once the columns of a previously formed section have all been marked. This is possible since the x_j values associated with the original section will

never be utilized; at no time will a column be reselected once it has been marked, either as a potential solution or as the min C_j column.

Several purely computer-related techniques were investigated in further attempts to decrease the core storage requirements of the program. Among these were the use of an overlay structure and the use of direct-access storage areas. Although an overlay structure[6] has been incorporated in the final program, its use accounts for only a small saving of space. The use of the direct access devices did decrease storage requirements considerably, but computation time increased on the average by a factor of six; this due primarily to the fact that the problems requiring the most space also require a considerable number of iterations, and prohibitively high direct access retrieval times for each iteration make the use of this technique highly inefficient.

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[6]Appendix B gives a brief discussion of the overlay technique utilized by the program.

IV. TEST PROBLEMS

Problems 1 through 9 were taken directly from a collection of twenty-five integer programming test problems compiled at Stanford University by John Haldi [4]. Of particular interest were problems 1, 3, 4, 5, 6, and 9. Upper bounds were computed for problems 1 and 3 using the "course" duplicate column elimination criteria, but the program ran out of space prior to reaching the optimal solution itself under the "fine" duplicate column elimination run. Problem 4 was a good test of how the program handles problems whose continuous solution includes non-basic variables at their upper bounds (i.e., the program allows negative C_j values as long as the upper bound constraints are maintained for the integer portion of the solution). Problem 9 provided an example of how the "course" duplicate column elimination run can bypass the optimal solution by eliminating (marking) all columns duplicating the one which produced an infeasible solution, regardless of their corresponding non-basic variable values.

Problems 5 and 6 are identical to problems 2 and 4 with the exception of one value in each. The Haldi paper gave optimal solutions for these two problems which did not satisfy the constraints as stated. However, with the modifications of problems 5 and 6, the answers given were both optimal and feasible.

Problem 10 is the example problem from the original paper by Dr. Greenberg [1], which demonstrates the use of the algorithm on bounded variable problems.

Problem 11 is a simple text book example [5] which depends heavily upon the algorithm modification requiring an immediate search for integer

solutions. This problem also appears in reference [8] to demonstrate an all integer algorithm by R. E. Gomory.

Problem 12 is an exercise from [9] which demonstrates the existence of a set of relatively small integer LP problems the solution of which can frequently be quite tedious and inefficient when employing R. E. Gomory's all integer algorithm [8]. Although it has been proven that this particular algorithm will converge to an integer solution in all cases, this convergence can be so slow as to require the use of an alternate all integer procedure such as the algorithm employed by the program presented here. (Note the relative ease with which the program handles solution of the problem).

Problem 13 was specifically developed to demonstrate how the program will handle a problem which has a feasible continuous solution but no feasible integer solution.

Except where specifically indicated, the variables are bounded below by zero and above by infinity and are all required to be integer.

A. SIX FIXED CHARGE PROBLEMS [4]:

Maximize $C^T X$

Subject to $F\delta(X) + A^T(X) \leq b$

$X_i \geq 0$

$$\delta(X) = \begin{cases} 0 & X_i = 0 \\ 1 & X_i > 0 \end{cases}$$

All problems are similar except for parameters a_1, a_2, and b_2 and all unsubscripted variables are vectors.

	F1	F2	A1	A2	A3	S1	S2	S3	S4
max	0	0	1	1	1	0	0	0	0

	F1	F2	A1	A2	A3	S1	S2	S3	S4	
subject to										
	2	3	1	2	2	1	0	0	0	b_1
	3	2	2	1	2	0	1	0	0	b_2
	$-a_1$	0	0	0	0	0	0	1	0	
	0	$-a_2$	0	0	0	0	0	0	1	

Problem	a_1	a_2	b_1	b_2
1	6	7	18	15
2	9	7	18	17
3	9	9	21	21
4	6	8	19	25
5	7	7	18	17
6	6	8	19	15

B. THREE IBM TEST PROBLEMS [4]:

Problems 7 and 8 are identical with the exception of b. (Note:

(Note: slack variables must be added).

Min $\vec{c}'\vec{x}$

subject to

$$\vec{Ax} \geq \vec{b} \quad , \; x_j \geq 0$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
min	1	1	1	1	1	1	1
subject to:	1	0	0	1	1	0	1
	0	1	0	1	0	1	1
	0	0	1	0	1	1	1
	1	1	0	0	1	1	0
	1	0	1	1	0	1	0
	0	1	1	1	1	0	0
	1	1	1	0	0	0	1

Problem	b_1	b_2	b_3	b_4	b_5	b_6	b_7
7	5	5	5	4	4	4	3
8	4	4	4	3	3	3	2

Problem 9:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7		b
min	13	15	15	11	0	0	0		
subject to:									
	4	5	3	6	-1	0	0	=	96
	20	21	17	12	0	-1	0	=	200
	11	12	12	7	0	0	-1	=	101

C. MISCELLANEOUS PROBLEMS:

Problem 10:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8		b
min	5	7	10	3	1	0	0	0		
subject to:										
	1	-3	5	1	-4	-1	0	0	=	2
	-2	6	-3	-2	2	0	-1	0	=	0
		-2	2	-1	-1	0	0	-1	=	1

$$0 \leq x_j \leq 1 \qquad j = 1, \ldots, 5$$

Problem 11:

	x_1	x_2	x_3	x_4	x_5	x_6		b
min	10	14	21	0	0	0		
subject to:								
	8	11	9	-1	0	0	=	12
	2	2	7	0	-1	0	=	14
	9	6	3	0	0	-1	=	10

Problem 12:

	x_1	x_2	x_3	x_4	b
min	-1	-4	0	0	
subject to:					
	2	4	1	0	= 7
	10	3	0	1	= 15

Problem 13:

	x_1	x_2	x_3	x_4	x_5	x_6	b
min	5	10	0	0	0	0	
subject to:							
	10	10	1	0	0	0	= 55
	10	10	0	-1	0	0	= 45
	-10	10	0	0	1	0	= 5
	10	-10	0	0	0	1	= 5

V. COMPUTATIONAL RESULTS

Two factors significantly affect the success or failure of the program as it processes each problem. One of these is the spread between the maximum and minimum coefficients in the objective function of the equivalent problem. Should this spread be wide enough, it can easily be demonstrated that the columns corresponding to the one with the high objective function coefficient in subsequent sections may require an extremely large number of sections prior to being marked as the minimum C_j. This will require that each section be provided "fresh" section space in core (i.e., there will be no space available for reuse since no section will become fully marked until late in the computation), and that the program runs the high risk of running out of space prior to reaching a solution.

Another obstacle to successful problem solution is the size of the maximum period for each section. As described previously, the higher the period, the more the distinct column vectors of fractional parts which can be formed by the algorithm; hence, the column vector which produces the optimal solution very likely will be generated only after a large number of sections. Table I is a compilation of computational results from five additional problems which the program failed to solve.[7] The high section counts and corresponding high computation times make the use of an iterative scheme of the type employed by the algorithm very

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[7] These problems are listed as numbers 5 through 9 of the first 10 appearing in [4] and are of interest here for two reasons: none was larger than six constraints and 11 variables, and all had relatively high common denominators.

TABLE I

Additional Test Problem Results

PROBLEM	COMMON DENOMINATOR	SECTIONS[10]	APPROXIMATE TIME (SECS)
5 and 7[9]	19400	1744	651
		754	268
6 and 8[9]	32000	2591	969
		967	350
9	2000	1400	895
		878	486

PROBLEM	MAX C_j WHEN PROGRAM STOPPED[10]	SECTION #1 Min C_j	SECTION #1 Max C_j	C_j REQUIRED TO PRODUCE INTEGER SOLUTIONS[8]
5 and 7	59750	4000	9000	244600
	30450			
6 and 8	118000	1300	5550	398000
	56000			
9	5600	400	800	6000
	4400			

---

[8] Computed using the answers supplied in [4].

[9] Both problems are identical with one exception: the first has bounded variables and the second introduces two additional constraints to produce the bounds.

[10] The first figure represents values for the problem under the "course" duplicate column elimination method; the second figure represents values when run under the "fine" method. All timing figures are exclusive of input/output times. All problems stopped because the limitation of 450 section spaces was exceeded.

unattractive economically, at least for those problems whose common denominators cannot be reduced into the hundreds.[11]

In addition to the inefficiencies associated with problems with high common denominators, there also exists the possibility, albeit remote, of computer oriented round-off errors for these same problems. The round-off errors occur in the computation of the continuous solution basic variable values and equivalent problem coefficients and have the effect of changing a problem with a feasible integer solution to one with no feasible integer solution. This has occurred, however, in just two problems (listed in Table I), both of which had 32000 as their common denominators.[12] The particular error appeared as a miscalculation of 1/32000 in one fractional part value. No other problem listed in Table I experienced such an error.[13] Although these errors destroy any possibility of arriving at correct integer solutions, it would appear as if they occur only for extremely high common denominators and available data indicates that perhaps the iterative scheme should be abandoned for these problems in the first place.

In order to reduce total storage requirements by approximately one third, the size of the storage word for the fractional parts of the equivalent problem constraint coefficients and right-hand sides was reduced

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[11] By prior developments, the period is necessarily less than or equal to the common denominator; the highest period for the successful test problems was 320.

[12] The error was artificially corrected to produce the data for Table I.

[13] This fact was verified through the use of a computer program, developed by Dr. R. Shudde of the Naval Postgraduate School, which provides continuous results in integer format. The utilization of such a program will, of course, eliminate all possibility of round-off error.

by one-half. The particular procedure employed to effect this reduction is peculiar to the IBM System 360 and has the effect of limiting the values of equivalent problem common denominators to 32767; in light of the preceding discussion, this does not however appear to be too unreasonable a restriction.

TABLE II

Solutions

PROBLEM	Optimal Objective Function Value	OPTIMAL VARIABLE COSTS Subscripts 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1[14]	7	0	0	0	0	7	4	1	0	0					
2	8	0	0	0	0	8	2	1	0	0					
		1	1	3	5	0	0	1	6	2					
		1	1	4	4	0	1	0	5	3					
		1	1	3	4	1	0	0	6	3					
3[14]	10	1	1	4	6	0	0	2	5	3					
		1	1	5	5	0	1	1	4	4					
		0	0	0	0	10	1	1	0	0					
4	11	1	0	6	0	5	1	0	0	0					
		1	0	5	0	6	0	0	1	0					
5	8	0	0	0	0	8	2	1	0	0					
		1	1	3	5	0	0	1	4	2					
		1	1	4	4	0	1	0	3	3					
		1	1	3	4	1	0	0	4	3					
6	8	0	1	0	8	0	0	5	0	0					
		0	1	0	7	1	0	4	0	1					
		0	1	0	6	2	0	3	0	2					
		0	1	0	5	3	0	2	0	3					
		0	1	0	4	4	0	1	0	4					
		1	1	2	6	0	0	0	4	2					
		0	1	0	3	5	0	0	0	5					

[14]Time/space limitations were reached prior to solving for all alternative optimal solutions.

TABLE II— CONTINUED

PROBLEM	Optimal Objective Function Value	OPTIMAL VARIABLE COSTS Subscripts 1	2	3	4	5	6	7	8	9	10	11	12	13	14
		1	0	0	2	2	1	2	2	0	0	0	0	0	0
		0	1	0	2	1	2	2	0	2	0	0	0	0	0
		1	1	0	1	2	2	1	0	0	0	2	0	0	0
7	8	1	0	1	2	1	2	1	0	0	0	0	2	0	0
		0	1	1	2	2	1	1	0	0	0	0	0	2	0
		1	1	1	1	1	1	2	0	0	0	0	0	0	2
		1	0	0	2	2	0	2	3	0	0	0	0	1	1
		1	1	0	1	1	1	2	1	1	0	1	0	0	2
		1	0	1	1	1	1	2	1	0	1	0	1	0	2
		1	1	1	1	1	0	2	1	0	0	0	0	1	3
		0	1	1	1	1	1	2	0	1	1	0	0	1	2
		1	1	1	1	0	1	2	0	1	0	0	1	0	3
		1	1	1	0	1	1	2	0	0	1	1	0	0	3
		1	1	1	1	1	1	1	0	0	0	1	1	1	2
		0	0	0	2	1	2	2	1	2	1	0	1	0	0
		0	1	0	2	1	1	2	1	2	0	0	0	1	1
		0	0	0	1	2	2	2	1	1	2	1	0	0	0
		0	0	1	1	2	1	2	1	0	2	0	0	1	1
		1	0	0	1	2	2	1	1	0	1	2	1	0	0
		1	1	0	1	2	1	1	1	0	0	2	0	1	1
		1	0	1	2	1	1	1	1	0	0	0	2	1	1
		0	1	1	2	2	0	1	1	0	0	0	0	3	1
		0	1	0	2	0	2	2	0	3	0	0	1	0	1
		0	0	1	1	1	2	2	0	1	2	0	1	0	1
		1	1	0	1	1	2	1	0	1	0	2	1	0	1
8	7	0	1	0	1	2	2	1	0	1	1	2	0	1	0
		1	0	1	2	0	2	1	0	1	0	0	3	0	1
		0	0	1	2	1	2	1	0	1	1	0	2	1	0
		0	1	1	2	1	1	1	0	1	0	0	1	2	1
		0	0	1	0	2	2	2	0	0	3	1	0	0	1

TABLE II— CONTINUED

PROBLEM	Optimal Objective Function Value	OPTIMAL VARIABLE COSTS Subscripts 1	2	3	4	5	6	7	8	9	10	11	12	13	14
8	7	1	1	0	0	2	2	1	0	0	1	3	0	0	1
(Continued)		1	0	1	1	1	2	1	0	0	1	1	2	0	1
		1	1	0	1	2	2	0	0	0	0	3	1	1	0
		0	1	1	1	2	1	1	0	0	1	1	0	2	1
		1	0	1	2	1	2	0	0	0	0	1	3	1	0
		0	1	1	2	2	1	0	0	0	0	1	1	3	0
		0	0	0	2	2	1	2	2	1	1	0	0	1	0
		1	0	0	2	1	1	2	2	1	0	0	1	0	1
		1	0	0	1	2	1	2	2	0	1	1	0	0	1
		1	0	0	2	2	1	1	2	0	0	1	1	1	0
		1	0	0	2	1	2	1	1	1	0	1	2	0	0
		0	1	0	2	2	1	1	1	1	0	1	0	2	0
		0	0	1	2	2	1	1	1	0	1	0	1	2	0
		0	1	0	1	1	2	2	0	2	1	1	0	0	1
		0	1	0	2	1	2	1	0	2	0	1	1	1	0
		0	0	1	1	2	2	1	0	0	2	1	1	1	0
9	187	0	0	0	17	6	4	18							
10	17	0	1	1	0	0	0	3	0						
11	52	1	0	2	14	2	5								
12	5	5	1	1	1	2									
13		INFEASIBLE													

TABLE III

Sections Required and Computation Times

PROBLEM	COURSE[15]		FINE[16]	
	Sections[18]	Time[17]	Sections[18]	Time[17]
1	247	16.55	768+	272.06
2	253	21.75	769	287.48
3	340	28.20	967+	360.51
4	4	0.08	4	0.08
5	185	10.55	960	347.86
6	145	9.15	580	164.90
7	8	0.22	8	0.22
8	32	1.01	330	64.70
9	2000	416.40	412	65.60
10	5	0.06	5	0.06
11	39	0.49	39	0.49
12	3	0.01	3	0.01
13		INFEASIBLE		

[15]Refers to the modified duplicate column elimination method which will provide a bound for the optimal solution.

[16]Refers to the original duplicate column elimination method which ultimately provides the optimal solution.

[17]The the nearest 1/100 second; exclusive of input/output times and alternate optima requirements.

[18]Referred to as iterations in Table IV, this is new sections formed and not section space required; '+' means unsolved.

VI CONCLUSIONS

Table IV is a comparison of section[19] requirements of the program with three IP programs available through SHARE [4]. Although in general the SHARE programs require considerably fewer iterations, the computer program presented here has several rather attractive benefits associated with its use. As discussed previously, all integer solutions to the equivalent knapsack problem are systematically produced in order of increasing objective function value; it is the associated iterative scheme of solution which will provide all existing alternate optima to integer LP problems. Also, as alluded to in the preceding discussion of test problem 12, the basic algorithm is a good substitute for Gomory's all integer algorithm in the solution of at least some integer problems. The computer program presented here does, however, have definite limitations.

As mentioned in the preceding section, the size of the period, and hence the common denominator, plays a major role in the program requirements for core and time; it is these requirements which ultimately restrict the size of the problem and the number of sections which the program will accommodate. Since the only test problems which failed to produce at least one optimal solution were those whose common denominators ranged from 2000 to 32000, the question arises: can this common denominator be approximated (i.e., reduced) to a point at which corresponding periods, and hence time and space requirements, are no longer prohibitively high and at the same time solutions do not become distorted? The ultimate utility of the program rests on the answer to this question.

---

[19] Program iterations refer to sections formed during the "fine" duplicate column elimination run; iterations for the three SHARE programs mean total pivot steps required to solve the problem.

TABLE IV

A Comparison of Iteration Requirements

PROBLEM NUMBER	CONSTRAINTS	VARIABLES	PROGRAM	LIP1	IPOZ	IPO3
1	4	9	768+	25	19	51
3	4	9	967+	22	17	80
5	4	9	960	21	18	58
6	4	9	580	17	29	35
					IPMZ	IPM3
7	7	14	8	11	7	5
8	7	14	330	30	10	11
9	3	7	412	63	2	28

APPENDIX A

Input Data and Output Formats

Input[20]

Card 1,	cc 5:	number of constraints
	cc 10:	number of variables (including slack)
	cc 15:	1 if bounds are required
	cc 21-71:	Problem title, if any ("STOP" after last card of last problem deck)
	cc 72:	1 if any existing alternative optimal solutions are to be listed
Card Set 1:		These are the constraint coefficients, A, in the form A(I,J), where I is the constraint number, J the variable.
	cc 1:	1 if last card in the set
	cc 4:	constraint number
	cc 7:	variable number
	cc 12:	coefficient value
Card Set 2:		These are the right hand sides, H, in the form H(I), where I is the constraint number
	cc 1:	1 if last card in the set
	cc 4:	constraint number
	cc 9:	right hand side value

[20]All input defaults to zero (bounds to zero and infinity). All data will be right adjusted to the card column (cc) indicated.

Card Set 3: These are the objective function costs, A, in the form A(M,J), where J is the variable, M the number of constraints plus one (completed separately).

cc 1: 1 if last card in set

cc 4: variable number

cc 9: cost

Card Set 4: These are the upper bounds on the variables in the form XUB(J), where J is the variable.

cc 1: 1 if last card in set

cc 4: variable number

cc 9: variable bound

Output

The first series of lines provides a review of the input data, in its original format. The continuous solution is then listed with its computation time. Next, all intermediate feasible solutions are listed with their individual computation times, iterations required and total section spaces required. (If alternate optima are available, they will appear at this point also.) The first section in the integer solution and the common denominator are then listed to provide a feel for the iteration requirements of the particular problem. Finally, the last optimal solution and its time and iteration requirements are listed.

APPENDIX B

Overlay Structure[21]

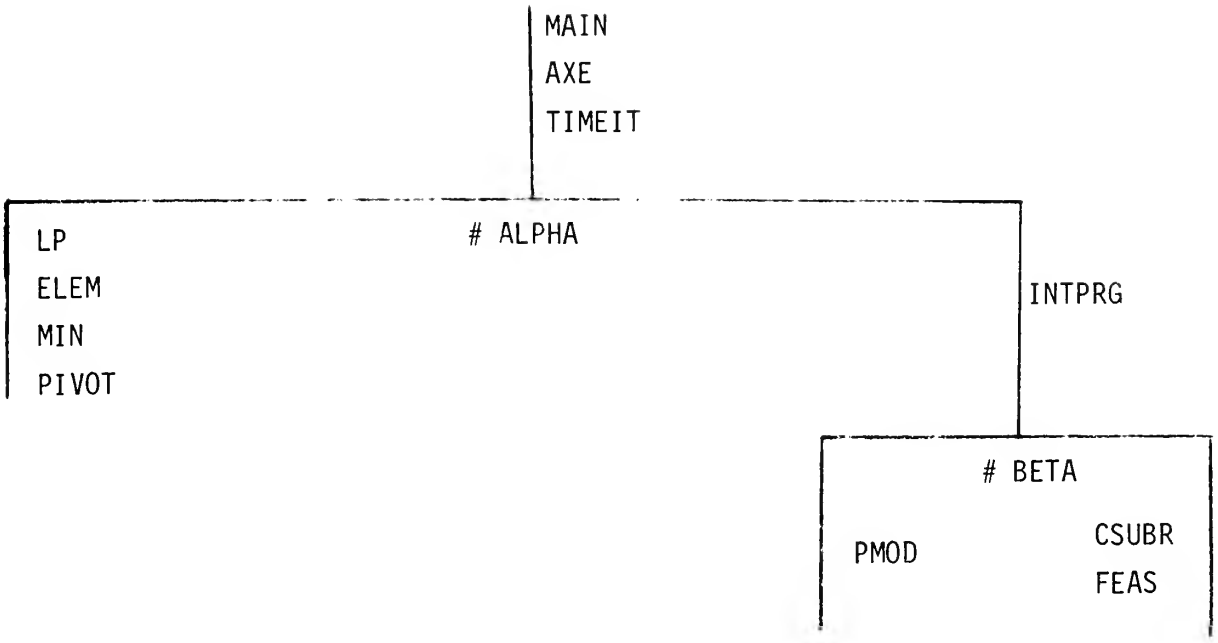
The program, with its present capacity, requires approximately 370 thousand bytes of storage in the IBM System/360-67 computer. By utilizing an overlay structure, this requirement can be reduced to approximately 360 thousand bytes. Although relatively small, this is as large a reduction as possible due to the high degree of interdependence among the subroutines in the program. Common blocks were specifically developed to take maximum advantage of the overlay structure.

The following cards, were placed directly behind the program in object deck form and were the product of the overlay tree of Figure 1.

```
OVERLAY ALPHA
INSERT LP, ELEM, PIVOT, MIN
OVERLAY ALPHA
INSERT INTPRG
OVERLAY BETA
INSERT PMOD
OVERLAY BETA
INSERT CSUBR, FEAS
```

---

[21] References [10] and [11] were utilized extensively in the development of the overlay structure and should be consulted for a more detailed explanation of the techniques employed by the program.



THE OVERLAY TREE

Figure 1.

```
C
C
C
C
C     ***********************************************
C     *                                             *
C     *                                             *
C     *      A COMPUTER PROGRAM FOR THE SOLUTION    *
C     *                                             *
C     *                TO INTEGER LINEAR            *
C     *                                             *
C     *             PROGRAMMING PROBLEMS            *
C     *                                             *
C     *                                             *
C     ***********************************************
C
C
C      THE SUBPROGRAMS IMMEDIATELY FOLLOWING 'MAIN' (LP,
C          ELEM, MIN, AXE, AND PIVOT) PROVIDE THE EQUIVALENT
C          PROBLEM (I.E., THE CONTINOUS SOLUTION TO THE
C          ORIGINAL PROBLEM).
C
C     THE FINAL FOUR SUBPROGRAMS OPERATE ON THE EQUIVALENT
C          PROBLEM, USING THE INTEGER ALGORITHM, TO PROVIDE
C          AN INTEGER SOLUTION TO THE PROBLEM.
C
C     PROGRAM CAPACITY WILL ALLOW PROBLEMS OF UP TO 10
C          CONSTRAINTS AND 22 VARIABLES.
C
C     A TIMING ROUTINE IS PROVIDED AND WILL INDICATE
C          INDIVIDUAL TIMES REQUIRED TO COMPUTE THE
C          CONTINUOUS SOLUTION AND THE INTEGER SOLUTION.
C
C     ALL ALTERNATIVE OPTIMA WILL BE PROVIDED WHEN SETTING
C          IALT = 1 (PUNCH '1' IN COL. 72 OF FIRST DATA CARD)
C          CURRENT SPACE/TIME LIMITATIONS WILL NORMALLY
C          PROVIDE A MINIMUM OF TWO OR THREE ALTERNATE
C          SOLUTIONS.
C
C
C     PROGRAM TO ENTER DATA AND CALL LP
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/TWO/Y(14),B(14,14),ISOL(14),ASAVE(14,24),
     1HSAVE(14)
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
C
C     PROGRAM REQUIRES DATA INPUT IN THE FORM OF A
C          MINIMIZATION PROBLEM
C
C     IGNAT=0 MEANS MINIMIZE PROBLEM
C     IGNAT=1 MEANS MAXIMIZE PROBLEM
C     MM = NUMBER OF EQUATIONS
C     N = NUMBER OF VARIABLES
C     BDN = 0 MEANS NO BOUNDS
C     BND = 1 MEANS BOUNDS
C     IALT = 1 MEANS ALTERNATIVE OPTIMA WILL BE PROVIDED
C
      DIMENSION RUN(12)
      INTEGER*4 RUN,STOPIT
      DATA STOPIT/'STOP'/
      INTEGER BND
 8338 IOT=0
      IOPP=0
      READ(5,203)MM,N,BND,IGNAT,RUN,IALT
  203 FORMAT(4I5,12A4,3X,I1)
      IF(RUN(1).EQ.STOPIT) STOP
```

```
      WRITE(6,204)RUN
  204 FORMAT(1H1,29X13A4)
      WRITE (6,2)MM,N
    2 FORMAT(//4I5)
      M = MM+1
      DO 1 I=1,M
      H(I)=0.
      DO 1 J=1,N
    1 A(I,J)=0.
      DO 9 J=1,N
      XUB(J)=-1.
    9 XLB(J)=0.
    3 READ (5,4)IL,I,J,A(I,J)
    4 FORMAT(I1,2I3,F5.0)
      IF(IL.EQ.0)GO TO 3
C
C     H(I) ARE THE RIGHT HAND SIDES,H(M) = INITIAL Z
C
    5 READ (5,6)IL,I,H(I)
    6 FORMAT (I1,I3,F5.0)
      IF(IL.EQ.0)GO TO 5
C
C     A(M,J) ARE THE COSTS
C
    7 READ (5,6)IL,J,A(M,J)
      IF(IL.EQ.0)GO TO 7
      IF(BND.EQ.0)GO TO 88
   11 READ(5,12)IL,J,XUB(J)
   12 FORMAT (I1,I3,F5.0)
      IF(IL.EQ.0)GO TO 11
C
C     INITIAL INPUT VALUES ARE SAVED FOR USE ON SECOND
C         ATTEMPT AT SOLUTION IF REQUIRED
C
   88 DO21I=1,MM
      DO21J=1,N
   21 ASAVE(I,J)=A(I,J)
      DO22I=1,M
   22 HSAVE(I)=H(I)
      DO23J=1,N
   23 ASAVE(M,J)=A(M,J)
      MSAVE=M
      MMSAV=MM
      NSAVE=N
      GOTO8
   55 N=NSAVE
      IOPP=0
      M=MSAVE
      MM=MMSAV
      DO24I=1,MM
      DO24J=1,N
   24 A(I,J)=ASAVE(I,J)
      DO25I=1,M
   25 H(I)=HSAVE(I)
      DO26J=1,N
   26 A(M,J)=ASAVE(M,J)
      WRITE(6,6000)
 6000 FORMAT('1')
    8 WRITE(6,10) ((A(I,J),J=1,N),I=1,MM)
      WRITE(6,10) (H(I),I=1,M)
      WRITE(6,10) (A(M,J),J=1,N)
      WRITE(6,10) (XUB(J),J=1,N)
      WRITE(6,10) (XLB(J),J=1,N)
   10 FORMAT(1H ,10F8.0)
      ICHK=0
C
C     TIMER STARTED FOR CONTINUOUS SOLUTION
C
      CALL TIMEIT( 0,TIMEX)
      CALLLP
      IF(ICHK.EQ.0)GOTO1063
C
```

```
C       TIMER STARTED FOR INTEGER SOLUTION
C
        CALL TIMEIT( 0,TIMEX)
        CALL INTPRG
        IF(IOT.EQ.1.AND.IOPP.EQ.1)GOTO222
        IF(IOT.EQ.1.AND.IOPP.NE.1)GOTO55
        GOTO1063
  222   WRITE(6,223)
  223   FORMAT(///' ABOVE SOLUTION(S) REPRESENT UPPER/LOWER BO
       1  OPTIMAL SOLUTION..PROBLEM NOW RERUN USING TOTAL DUP.
        WRITE(6,224)
  224   FORMAT(/' IF THIS METHOD FAILS OR IS INCOMPLETE DUE TO
       1LIMITATIONS, THIS/THESE BOUNDS IS/ARE THE BEST APPROXI
        WRITE(6,225)
  225   FORMAT(/' TO A SOLUTION WHICH THE PROGRAM CAN COMPUTE.
        GOTO55
 1063   WRITE(6,14)
   14   FORMAT(1HO,10X, ' HAVE FINISHED ')
        GOTO8888
        END
```

```
      SUBROUTINELP
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/TWO/Y(14),B(14,14),ISOL(14),ASAVE(14,24),
     1HSAVE(14)
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
      IW=0
      Z=H(M)
      L=0
      M=MM+2
      ID=0
      DO 3 J=1,MM
      DO 2 I=1,M
    2 B(I,J)=0.
      B(M,J)=-1.
    3 B(J,J)=1.
C     XI(J)=+1. MEANS INCREASE
C     XI(J)=-1. MEANS DECREASE
C     XI(J)=2.  MEANS BASIC
C     XI(J)=0. MEANS NONBASIC
      DO 24 J=1,N
      XI(J)=0.
   24 X(J)=0.
      DO 4 I=1,MM
      Y(I)=H(I)
    4 IB(I)=-I
      AD=1.
      DI=1.
      E=.5
      W=0.
      DO 5 J=1,N
      A(M,J)=0.
      DO 5 I=1,MM
    5 A(M,J)=A(M,J)-A(I,J)
      DO 6 I=1,MM
    6 W=W+H(I)
      ITR=0
      ITER=0
    8 JS=0
      DO 9 J=1,N
      IF(XI(J).EQ.2.) GO TO 9
      D=0.
      DO 7 K=1,MM
    7 D=D+B(M,K)*A(K,J)
      IF(W.EQ.0.) D=D+A(M,J)
      IF(ABS(D).LE.E)GO TO 9
      IF(D)15,9,16
   15 IF(XUB(J).EQ.-1.)GO TO 19
      IF(ABS(XUB(J)-X(J)).LT.E)GO TO 9
C     INCREASE
   19 XI(J)=1.
      GO TO 17
   16 IF(ABS(X(J)-XLB(J)).LT.E)GO TO 9
C     DECREASE
      XI(J)=-1.
   17 IF(JS.GT.0)GO TO 10
   61 JS=J
      DJS=D*XI(J)
      GO TO 9
   10 IF(DJS-D*XI(J).GE.E)GO TO 61
    9 CONTINUE
      IF(JS.EQ.0)GO TO 101
  221 CALLELEM
      ITER=ITER+1
      IF(IR.GT.0)GO TO 36
      IF(BND.EQ.0)GO TO 33
      IF(XH.EQ.1000000.)GO TO 33
```

```
   36 DO 38 I=1,MM
      IF(IB(I))39,528,40
   39 Y(I)=Y(I)-AX(I)*XH*XI(JS)
      GO TO 38
   40 K=IB(I)
      X(K)=X(K)-AX(I)*XH*XI(JS)
   38 CONTINUE
      X(JS)=X(JS)+XH*XI(JS)
      IF(W.EQ.0.)GO TO 62
      W=W+AX(M)*XH*XI(JS)
   62 Z=Z+AX(MM+1)*XH*XI(JS)
      IF(IR)528,50,44
   44 CALL PIVOT
   50 IF(W.GT.E)GO TO 8
      IF(IW.EQ.1)GO TO 8
      W=0.
      M=M-1
      IW=1
      GO TO 8
   33 CALLTIMEIT(-1,TIMEX)
      WRITE(6,542)TIMEX
  542 FORMAT(//' TIME TO COMPUTE CONTINUOUS SOLUTION = ',
     1-6PF15.6,' SECONDS.')
      WRITE(6,704)
  704 FORMAT(1H0,10X,18HUNBOUNDED SOLUTION)
      GO TO 528
  101 IF(W.GT.E)GO TO 35
      CALLTIMEIT(-1,TIMEX)
      WRITE(6,542)TIMEX
      WRITE(6,204)
  204 FORMAT(1HC,5X,6HRESULT)
      DO 31 J=1,N
      IF(X(J).LT.E)GO TO 31
      WRITE(6,200)J,X(J)
  200 FORMAT(1H ,5X,2HX(,I2,2H)=,F10.4)
   31 CONTINUE
      Z=SIGN(AINT(DI*ABS(Z)+.5)/DI,Z)
      WRITE(6,527)Z
  527 FORMAT(1HC,5X,2HZ=,F10.5)
C     HAVE CONTINUOUS SOLUTION
      IF(DI-1.)601,601,603
  601 CALLTIMEIT(-1,TIMEX)
      WRITE(6,542)TIMEX
      WRITE(6,701)
  701 FORMAT(1HC,5X,13HHAVE INTEGERS)
      DO45J=1,N
   45 ISOL(J)=X(J)+.5
      DO47J=1,N
      IF(ISOL(J))48,47,48
   48 WRITE(6,49)J,ISOL(J)
   49 FORMAT(1H0,5X,2HX(,I2,2H)=,I8)
   47 CONTINUE
      GOTO528
  603 IF(ABS(ABS(Z)-AINT(ABS(Z)+E)).GE.E)GO TO 6032
      DO 637 I=1,MM
      IF(IB(I))637,637,83
   83 J=IB(I)
      IF(ABS(X(J)-AINT(X(J)+E))-E)637,637,6032
  637 CONTINUE
C     HAVE ALL INTEGERS
      WRITE(6,701)
      GOTO528
   35 CALLTIMEIT(-1,TIMEX)
      WRITE(6,542)TIMEX
      WRITE(6,205)
  205 FORMAT(1HC,' INFEASIBLE ')
      GOTO528
 6032 ICHK=1
      WRITE(6,703)
  703 FORMAT(' HAVE FRACTIONS')
  528 RETURN
      END
```

```
      SUBROUTINEELEM
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/TWO/Y(14),B(14,14),ISOL(14),ASAVE(14,24),
     1HSAVE(14)
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
      IR=0
      CALLAXE
      EE=E
      IF(BND.EQ.0.)GO TO 12
      IF(XI(JS).EQ.1.)GO TO 1
      GO TO 2
    1 XH=XUB(JS)-X(JS)
      IF(XUB(JS).EQ.-1.)XH=1000000.
      GO TO 12
    2 XH=X(JS)-XLB(JS)
   12 DO 25 I=1,MM
      IF(BND.GT.0.)GO TO 810
      IF(AX(I).LE.E)GO TO 25
      AH=AX(I)
      HH=H(I)
      IF(IR)25,811,812
  811 IR=I
      XH=HH/AH
      EE=E/AH
      GO TO 25
  812 CALL MIN (I)
      GO TO 25
  810 IF(ABS(AX(I)).LT.E)GO TO 25
      IF(XI(JS).EQ.1.)GO TO 8
C     DECREASE
      IF(IB(I))3,25,4
    4 K=IB(I)
      IF(AX(I))5,25,6
    6 IF(XUB(K).EQ.-1.)GO TO 25
      HH=XUB(K)-X(K)
      AH=-AX(I)*XI(JS)
      CALL MIN (I)
      GO TO 25
    5 HH=X(K)-XLB(K)
      AH=AX(I)*XI(JS)
      CALL MIN (I)
      GO TO 25
    3 IF(AX(I))15,25,16
   16 HH=Y(I)
      AH=-AX(I)*XI(JS)
      CALL MIN (I)
      GO TO 25
   15 HH=Y(I)
      AH=AX(I)*XI(JS)
      CALL MIN (I)
      GO TO 25
C     INCREASE
    8 IF(IB(I))30,25,40
   40 K=IB(I)
      IF(AX(I))6,25,5
   30 IF(AX(I))16,25,15
   25 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MIN (I)
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
      IF(ABS(HH-AH*XH)-EE)70,70,2
    2 IF(HH-AH*XH)24,70,25
   24 XH=HH/AH
    3 IR=I
      EE=E/AH
      GO TO 25
   70 IF(IB(I))74,25,73
   73 IF(IB(IR))25,25,4
   74 IF(IB(IR))4,25,3
    4 IF(ABS(AX(IR))-ABS(AX(I)))25,25,3
   25 RETURN
      END
```

```
      SUBROUTINEAXE
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/TWO/Y(14),B(14,14),ISOL(14),ASAVE(14,24),
     1HSAVE(14)
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
      DO 21 I=1,M
      AX(I)=0.
      DO 21 K=1,MM
   21 AX(I)=AX(I)+B(I,K)*A(K,JS)
      AX(MM+1)=AX(MM+1)+A(MM+1,JS)
   22 RETURN
      END
```

```
      SUBROUTINE PIVOT
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/TWO/Y(14),B(14,14),ISOL(14),ASAVE(14,24),
     1HSAVE(14)
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
      AD=AD*ABS(AX(IR))
      DI=AINT(DI*ABS(AX(IR))+.5)
   84 IF(IB(IR))1,1,2
    2 K=IB(IR)
      XI(K)=0.
    1 IB(IR)=JS
      XI(JS)=2.
      E=.5/DI
      DO 29 I=1,M
      IF(I.EQ.IR)GO TO 29
      P=-AX(I)/AX(IR)
      H(I)=H(I)+P*H(IR)
      DO 31 J=1,MM
   31 B(I,J)=B(I,J)+P*B(IR,J)
   29 CONTINUE
      P=1./AX(IR)
      H(IR)=H(IR)*P
      DO 33 J=1,MM
   33 B(IR,J)=B(IR,J)*P
      DO 90 K=25,600,25
      IF(K-ITR)90,91,95
   90 CONTINUE
      GO TO 95
   91 DO 92 I=1,M
      H(I)=SIGN(AINT(DI*ABS(H(I))+.5)/DI,H(I))
      DO 92 J=1,MM
   92 B(I,J)=SIGN(AINT(DI*ABS(B(I,J))+.5)/DI,B(I,J))
   95 ITR=ITR+1
      RETURN
      END
```

```
      SUBROUTINE INTPRG
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
C
C
C     VARIABLES
C          AT MATRIX
C             ROW 1 = EQUIVALENT OBJECTIVE FUNCTION
C                     COEFFICIENTS
C             ROW 2 = EQUIVALENT CONSTRAINT COEFFICIENTS
C             LAST COLUMN = EQUIVALENT RIGHT HAND SIDES
C          IX MATRIX
C             ROW 1 = NON-BASIC VARIABLE INDICES
C             ROW 2 = DUMMY (ORIGINALLY OBJECTIVE FUNCTION
C                     NUMERATORS)
C             ROW 3(FF) = CONSTRAINT COEFFICIENT FRACTIONAL
C                         PARTS
C             LAST ROW = MODIFIED NON-BASIC VARIABLE VALUES
C          IXX = OBJECTIVE FUNCTION NUMERATORS
C          IOPP=1 MEANS OPTIMAL SOLUTION REACHED
C          LIP = NUMBER OF POSSIBLE SOLUTIONS RECORDED
C          LT = ITERATION CURRENTLY BEING PROCESSED
C          IOT=1 RESULTS IN A RERUN OF THE PROBLEM AFTER
C                EITHER 450 SECTION SPACES HAVE BEEN USED OR
C                2000 ITERATIONS HAVE BEEN PERFORMED UNDER
C                THE MODIFIED DUPLICATE COLUMN ELIMINATION
C                RULE
C          IOT=2 MEANS THAT AFTER BOTH RUNS, SPACE AND
C                ITERATION LIMITS HAVE BEEN EXCEEDED AND THE
C                PROGRAM STOPS.
      COMMON MM,N,M,JS,E,IR,AD,W,Z,IM,DI,ITR,XH,AH,HH,EE,RL,
     1XT,XK,ID,BND,L,IW,ICHK,IGNAT,IOT,IOPP,IALT
C
C          IPP=1 VALUE OF CURRENT MIN C(J) PRODUCING A
C                POSSIBLE SOLUTION
C          IBJAY= OLD MIN C(J) TO BE RESTORED TO ICJAY IF NEW
C                 SOLUTION TRIED IS INFEASIBLE
C                 OR ITERATIONS EXCEED 2000,AND THE PROBLEM
C                 IS THE RERUN USING A COMPLETE  DUPLICATE
C                 CHECK
C
C          NB(I)= NON-BASIC VARIABLE BOUNDS
C          KB(I)=BASIC VARIABLE BOUNDS
C          KBVAR(I)= BASIC VARIABLE INDICES
C          LK =COMMON DENOMINATOR FOR MODIFIED COEFFICIENT
C          IG MATRIX
C             OUTPUT OF PMOD - FRACTIONAL PARTS OF AT MATRIX
C                              COMPONENTS
C          NBV= NUMBER OF NON-BASIC VARIABLES
C          IBV= NUMBER OF BASIC VARIABLES
C          N= NUMBER OF VARIABLES
      COMMON/FOUR/LT,IA,JA,IT,NBV,LK,IPP,IAT,JAT,ICJAY,IOP,
     1IBJAY,YLK,LKTT,LAP,KAT,IBV,LIP,IOU,MCR,ISUM,ICOST
     1,JUMP,LOKP,LOKPT,NCJAY,MCJAY
C          M= NUMBER OF BASIC VARIABLES +1
C          MM= NUMBER OF CONSTRAINTS
C          MB(I)= NON-BASIC VARIABLE INDICES (VALUES UNVARIED
C          KEY=1 MEANS A POSSIBLE SOLUTION HAS BEEN FOUND
C          IOP=1 MEANS A POSSIBLE SOLUTION WAS FOUND
C              INFEASIBLE AND SEARCH MUST CONTINUE
C          LAP=1 MEANS A TOTALLY MARKED SECTION IS TO PROVIDE
C              THE SPACE FOR A NEW SECTION
C          KAT= THE VALUE WHICH SAVES THE TOTAL NUMBER OF
C              SECTIONS AT THE TIME THE SECTION SPACE IS TO
C               BE RE-USED
C          LKTT= ITERATION COUNT
C          NB(J)=NON-BASIC VARIABLE BOUNDS
C          JUMP=1 MEANS THERE IS NO FEASIBLE SOLUTION TO THE
```

```
C                PROBLEM
C           ISUM = COMPUTED MAXIMUM POSSIBLE C(J), AFTER WHICH
C                  ALL COLUMNS OF FRACTIONAL PARTS WILL BE
C                  DUPLICATES
C
C
C
      COMMON/THREE/KBVAR(10),KB(10),MX(13),MB(23),NB(10),
     1AT(23,23),JX(13),AMX(13),MOX(22),IP(13,23),IXX(23,450)
     1,IIIP(22),NG(13),IX(13,23,450),IG(23,23)
      INTEGER*2 IX,IG
C
C
C     VALUES FROM CONTINUOUS SOLUTION ARE IDENTIFIED AND
C         ORDERED
C
C
C     NON-BASIC VARIABLE INDICIES AND BOUNDS
C
      NOTS=N-MM+1
      NOT=0
      DO1311J=1,N
      XTE=X(J)-XUB(J)
      YTE=ABS(XTE)
      IF(X(J).GT.E.AND.YTE.GT.E)GOTO1311
      NOT=NOT+1
      IX(1,NOT,1)=J
      IF(XUB(J).EQ.-1)GOTO1322
      NB(NOT)=XUB(J)
      GOTO1311
 1322 NB(NOT)=999999
 1311 CONTINUE
      IBV=MM
      KIBV=IBV+1
      AT(1,NOTS)=-H(KIBV)
      NBV=N-IBV
      JA=NBV+1
      IA=IBV+3
C
C     NON-BASIC VARIABLE VALUES INITIALIZED
C
      DO156I=1,JA
  156 IX(IA,I,1)=0
      NOTP=NBV+1
      IX(1,NOTP,1)=0
      M=MM+1
      LK=DI
C
C     DIAGNOSTIC MESSAGE PRODUCED WHEN THE COEFFICIENT
C         MATRIX COMMON DENOMINATOR EXCEEDS THE CAPACITY FOR
C         INTEGER*2 (32767), A SPACE SAVING DEVICE TO
C         INCREASE PROGRAM CAPACITY.  I.E., THE FRACTIONAL
C         PARTS OF THE COEFFICIENTS (OBJECTIVE FUNCTION
C         COEFFICIENTS ARE HANDLED SEPARATELY) COULD EXCEED
C         INTEGER*2 CAPACITY.
C
      IF(LK.LE.32767)GOTO1156
      WRITE(6,3333)LK
 3333 FORMAT(////' PROGRAM TERMINATED DUE TO POSSIBLE OVERFL
     1Y A HIGH COEFF. MAT. COMMON DENOMINATOR. (LK = ',I6,')
      STOP
C
 1156 YLK=0.0
      JSS=0
      DO9995JS=1,N
      CALL AXE
      DO9997I=1,IBV
      IF(JS.EQ.IB(I))GOTO9998
 9997 CONTINUE
      JSS=JSS+1
C
C     BASIC VARIABLE VALUES
```

```
C
      DO9996JST=2,M
      JSTT=JST-1
 9996 AT(JST,JSS)=AX(JSTT)
C
C     OPTIMAL CONTINUOUS SOLUTION (OR EQUIVALENT VALUE WHEN
C         A NON-BASIC VARIABLE IS AT ITS UPPER BOUND)
C
      AT(1,JSS)=AX(M)
C
      GOTO9995
 9998 NGO=0
      DO9991I=1,MM
      AXB=AX(I)-1.
 9991 IF(ABS(AXB).LE.E)NGO=I
 9992 KBVAR(NGO)=JS
      NGOO=NGO+1
      AT(NGOO,NOTP)=H(NGO)
      IF(XUB(JS).EQ.-1)GOTO55
      KB(NGO)=XUB(JS)
      GOTO9995
   55 KB(NGO)=999999
 9995 CONTINUE
      N=N-M+2
  503 IAT=IA-1
      JAT=JA-1
  668 DO33I=1,N
   33 MB(I)=IX(1,I,1)
      ICJAY=2147483646
      IBJAY=2147483646
      MCJAY=2147483646
      NCJAY=2147483646
C
C     COMPUTATION OF COEFFICIENT MATRIX FRACTIONAL PARTS
C         (I.E., MOD LK)
C
      CALL PMOD(M,N)
C
       DO 6 I=2,M
       DO 6 J=1,N
      NO=I+1
    6 IX(NO,J,1)=IG(I,J)
C
C     COMPUTATION OF NUMERATOR VALUES OF EQUIVALENT
C         OBJECTIVE FUNCTION
C
      LOKP=0
      LOKPT=2147483646
       DO 8 I=1,JA
      TLK=LK
      TALK=AT(1,I)*TLK
      BALK=TALK+.5
      IF(AT(1,I).LT.0)BALK=TALK-.5
      IX(2,I,1)=0
      IXX(I,1)=BALK
      IF(IXX(I,1).LE.LOKPT)LOKPT=IXX(I,1)
    8 IF(LOKPT.LT.0)LOKP=1
C
C
C     MAXIMUM POSSIBLE C(J) IS COMPUTED FOR USE IN
C         IDENTIFYING INTEGER PROBLEMS WITH NO FEASIBLE
C         INTEGER SOLUTION.
C
      ALOCK=LK-1
      ISQ=SQRT(ALOCK)
      DO667J=1,JAT
      DO670I=3,IAT
      DO6668L=1,ISQ
      LWW=L-1
      LW=ISQ-LWW
      PLW=LW
      APR=IX(I,J,1)
```

```
      PLK=LK
      ALL=APR/PLW
      ALLL=PLK/PLW
      ILL=ALL
      ILLL=ALLL
      CLL=ILL
      CLLL=ILLL
      BLL=ALL-CLL
      BLLL=ALLL-CLLL
      IF(BLL.EQ.0..AND.BLLL.EQ.0.)GOTO 670
 6668 CONTINUE
  670 NG(I)=LW
      KG=999999
      DO669K=3,IAT
  669 IF(NG(K).LE.KG)KG=NG(K)
  667 IIIP(J)=LK/KG
      ISUM=0
      DO671J=1,JAT
      IIIIP=IIIP(J)-1
      IXXTT=IXX(J,1)
      IF(IXX(J,1).LT.0)IXXTT=0
      ICOST=IIIIP*IXXTT
  671 ISUM=ISUM+ICOST
      JUMP=0
C
      LIP=0
      LT=1
      IOP=0
      LAP=0
      KAT=0
      LKTT=1
      IOU=0
   45 IPP=0
      LP=LT
      DO11LODE=LP,3000
      IF(IPP.EQ.1)GOTO200
C
C     SECTIONS ARE OPERATED ON TO IDENTIFY POSSIBLE
C         SOLUTIONS AND FORM NEW SECTIONS IF REQUIRED
C
      CALL CSUBR(N,IR,IGNAT,IOT,IOPP,IALT)
C
      IF(JUMP.EQ.1)GOTO173
      IF(IOPP.EQ.1)GOTO311
      IF(IOT.EQ.1.AND.IOU.EQ.1)GOTO504
      IF(IOT.EQ.2)GOTO1056
   11 CONTINUE
C
C     POSSIBLE SOLUTIONS CHECKED FOR FEASIBILITY AND EITHER
C         RECORDED OR DISCARDED
C
  200 CALL FEAS (IR,M,N,IGNAT)
C
      GOTO45
C
C     FINAL SOLUTION AND INITIAL SECTION ARE PRINTED
C
  311 CALLTIMEIT(-1,TIMEX)
      IF(IALT.EQ.1.AND.MCR.GE.ICJAY.AND.IOT.EQ.2)WRITE(6,889
  889 FORMAT(////' ALL EXISTING ALTERNATE OPTIMA ARE INCLUDE
     1DIATE SOLNS. FOR WHICH OBJ.FN. VALUES = FINAL SOLN VAL
      IF(IOT.EQ.1.AND.IALT.EQ.1)GOTO505
 1056 WRITE(6,102)
  102 FORMAT('1'////'
     1         SECTION #1')
      DO1057I=1,JAT
 1057 IX(1,I,1)=MB(I)
      DO104I=1,IA
      IF(I.EQ.2)GOTO144
      WRITE(6,103)(IX(I,J,1),J=1,JA)
      GOTO104
  144 WRITE(6,103)(IXX(JO,1),JO=1,JA)
```

```
  104 CONTINUE
  103 FORMAT(//1X10I12)
      WRITE(6,101)LK
  101 FORMAT(    ////' COMMON DENOMINATOR USED TO COMPUTE FRA
     1S = ',I7)
      IF(JUMP.EQ.1)GOTO504
      IF(IOT.EQ.2.AND.LIP.EQ.0)GOTO1050
      WRITE(6,400)
  400 FORMAT('1'////'
     1       OPTIMAL SOLUTION')
      WRITE(6,401)LKTT,LT
  401 FORMAT(///' TOTAL INTEGER ITERATIONS = ',I10,'.  TOTAL
     1CES REQUIRED = ',I4,', OF 450 AVAILABLE.')
      WRITE(6,3004)
 3004 FORMAT(//' NON BASIC VARIABLES INTEGER SOLUTION VALUES
      DO212I=1,JAT
      WRITE(6,3002)MB(I),MOX(I)
 3002 FORMAT(/' X( ',I2,' )=',I6)
  212 CONTINUE
      DO66I=1,IBV
      LZ=I+2
   66 WRITE(6,300)KBVAR(I),MX(LZ)
  300 FORMAT(//' INTEGER SOLN. VALUE FOR BASIC VARIABLE, X('
     1)
      WRITE(6,301) MX(2)
  301 FORMAT(////' OPTIMAL OBJECTIVE FUNCTION INTEGER SOLUTI
  505 WRITE(6,542)TIMEX
  542 FORMAT(//' COMPUTATION TIME SINCE LAST POSSIBLE INTEGE
     1',-6PF15.6,' SECONDS.')
      GOTO504
 1050 WRITE(6,1055)LKTT
 1055 FORMAT(//' SECTIONS REQUIRED EXCEEDS 450. TOTAL ITERAT
     1R = ',I10)
      GOTO504
  173 WRITE(6,174)ISUM,LKTT
  174 FORMAT(////' UPPER BOUND ON COST,',I10,', REACHED..ALL
     1LUMNS FORMED..NO FEASIBLE INTEGER SOLUTION EXISTS..ITE
     1I6)
      GOTO311
  504 RETURN
       END
```

```
      SUBROUTINE PMOD(M,N)
C
C
C     PROGRAM TO COMPUTE FRACTIONAL PARTS OF THE EQUIVALENT
C         PROBLEM CONSTRAINT EQUATIONS USING THE COMMON
C         DENOMINATOR, LK.
C
C     ELEMENTRY ARITHMETIC PROCESSES AND ROUNDING
C         TECHNIQUES UTILIZED
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/THREE/KBVAR(10),KB(10),MX(13),MB(23),NB(10),
     1AT(23,23),JX(13),AMX(13),MOX(22),IP(13,23),IXX(23,450)
     1,IIIP(22),NG(13),IX(13,23,450),IG(23,23)
      INTEGER*2 IX,IG
      COMMON/FOUR/LT,IA,JA,IT,NBV,LK,IPP,IAT,JAT,ICJAY,IOP,
     1IBJAY,YLK,LKTT,LAP,KAT,IBV,LIP,IOU,MCR,ISUM,ICOST
     1,JUMP,LOKP,LOKPT,NCJAY,MCJAY
        DO 12I=2,M
        DO 12J=1,N
      IF(AT(I,J).EQ.0.0)GOTO10
       GOTO11
   10  G=0.0
       GOTO1
   11 IF(AT(I,J).GT.0.0)GOTO2
      B=AT(I,J)-.5
       GOTO3
    2 B=AT(I,J)+.5
    3 IZ=B
      R=IZ
      IF(ABS(AT(I,J)).LE.ABS(R))IZ=AT(I,J)
      R=IZ
      IF(AT(I,J).LT.0.0)GOTO4
      EX=AT(I,J)-R
       GOTO5
    4 D=AT(I,J)+ABS(R)
       IF(D.EQ.0.0)GOTO8
       GOTO9
    8 G=0.0
      GOTO1
    9 EX=1.0+D
    5 F=EX*LK
      G=F+.5
    1 IG(I,J)=G
      IF(IG(I,J).EQ.LK)IG(I,J)=0
   12  CONTINUE
       RETURN
       END
```

```
      SUBROUTINE CSUBR(N,IR,IGNAT,IOT,IOPP,IALT)
C
CC
CC      PROGRAM TO DETERMINE MINIMUM C(J), TEST FOR POSSIBLE
C           SOLUTIONS, AND CONSTRUCT NEW SECTIONS
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/THREE/KBVAR(10),KB(10),MX(13),MB(23),NB(10),
     1AT(23,23),JX(13),AMX(13),MOX(22),IP(13,23),IXX(23,450)
     1,IIIP(22),NG(13),IX(13,23,450),IG(23,23)
      INTEGER*2 IX,IG
      COMMON/FOUR/LT,IA,JA,IT,NBV,LK,IPP,IAT,JAT,ICJAY,IOP,
     1IBJAY,YLK,LKTT,LAP,KAT,IBV,LIP,IOU,MCR,ISUM,ICOST
     1,JUMP,LOKP,LOKPT,NCJAY,MCJAY
      KEY=0
C
      IF(IOP.EQ.0)GOTO914
C
CC      A POSSIBLE SOLUTION WAS FOUND INFEASIBLE AND THE
C           PREVIOUS MIN C(J) MUST BE RESTORED
C
      IX(1,IR,IT)=0
      ICJAY=IBJAY
      IOP=0
      NCJAY=MCJAY
C
CC      NEWLY FORMED SECTION IS CHECKED FOR POSSIBLE SOLUTIONS
C
  914 DO88J=1,JAT
      IF(IX(1,J,LT).LE.0)GOTO88
      IF(IXX(J,LT).GT.NCJAY)GOTO89
      IF(IXX(J,LT).GT.ICJAY.AND.IXX(J,LT).LE.NCJAY.AND.LOKP.
     1EQ.1)GOTO88
      IF(IXX(J,LT).GT.ICJAY.AND.LOKP.NE.1)GOTO89
      DO8I=3,IAT
      RLK=(IX(I,J,LT)-IX(I,JA,1))
      IF(ABS(RLK).LE.YLK)GOTO8
      GOTO88
    8 CONTINUE
      IBJAY=ICJAY
      MCJAY=NCJAY
      ICJAY=IXX( J,LT)
      NCJAY= IXX(J,LT)
      IF(LOKP.EQ.1)NCJAY=IXX(J,LT)-LOKPT
      IR=J
      KEY=1
      GOTO188
   89 IX(1,J,LT)=0
   88 CONTINUE
      GOTO511
  188 IT=LT
      IX(1,IR,IT)=-1
      DO435I=1,IA
      DO435J=1,JA
      IP(I,J)=IX(I,J,IT)
      IF(I.EQ.2)IP(I,J)=IXX(J,IT)
  435 CONTINUE
      IF(LAP.NE.1)GOTO50
C
CC      'USED SPACE' KEYS INITIALIZED
C
      LAP=0
      LT=KAT
      KAT=0
      GOTO50
  511 IF(LAP.NE.1)GOTO512
      LAP=0
      LT=KAT
```

```
      KAT=0
C
C     MIN C(J) DETERMINED
C
  512 MCR=2147483646
      DO1L=1,LT
      DO1J=1,JAT
      IF(IX(1,J,L).EQ.0)GOTO1
      IF(MCR.GE.IXX( J,L))MCR=IXX( J,L)
    1 CONTINUE
C
C     SECTION CONTAINING MIN C(J) IDENTIFIED AND COLUMN
C          MARKED BY SETTING FIRST ROW VALUE TO ZERO
C
C          IT=SECTION IN WHICH MIN C(J) IS FOUND
C          IR=COLUMN CONTAINING MIN C(J)
C
      DO6I=1,LT
      IT=I
      DO6J=1,JAT
      IR=J
      IF(IX(1,J,I).EQ.0)GOTO6
      IF(MCR.EQ.IXX( J,I))GOTO7
    6 CONTINUE
    7 IX(1,IR,IT)=0
C
C     IF MIN C(J) IS GREATER THAN CURRENT VALUE OF C(J)
C          WHICH PRODUCED A FEASIBLE SOLUTION,THEN CURRENT
C          SOLUTION IS ALSO OPTIMAL AND PROBLEM IS SOLVED
C
      IF(IALT.EQ.1.AND.MCR.GE.ICJAY)GOTO888
      IF(IALT.EQ.1)GOTO557
      IF(IXX( IR,IT).GE.ICJAY)GOTO888
C
C
C
C     ALL SECTIONS CHECKED FOR FREE SPACE (I.E., ALL
C          COLUMNS MARKED)
C
  557 IF(LT.EQ.1)GOTO69
      DO66I=2,LT
      DO67J=1,JAT
      IF(IX(1,J,I).NE.0)GOTO66
   67 CONTINUE
      GOTO68
   66 CONTINUE
      GOTO69
C
C     A TOTALLY MARKED SECTION HAS BEEN FOUND AND ITS
C          LOCATION WILL BE USED BY THE NEW SECTION TO BE
C          FORMED
C
   68 KAT=LT
      LT=I
      LKTT=LKTT+1
      LAP=1
      GOTO70
C
C     NO TOTALLY MARKED SECTION HAS BEEN FOUND AND THE TOTAL
C          SECTION COUNT IS INCREMENTED BY ONE
C
   69 LT=LT+1
      LKTT=LKTT+1
   70 IF(LKTT.GE.2000.AND.IOT.NE.1)GOTO171
      IF(LT.GE.451)GOTO171
C
C
C     NEW SECTION FORMED
C
C
      DO 450 I=1,IA
      DO 450 J=1,JA
```

```
      IP(I,J)=IX(I,J,IT)
      IF(I.EQ.2)IP(I,J)=IXX(J,IT)
  450 CONTINUE
      IX(IA,IR,LT)=IP(IA,IR)+1
      DO1111I=1,JA
 1111 IX(1,I,LT)=MB(I)
C
C     C(J) FORMED , CHECKED FOR GREATER THAN CURRENT C(J)
C          WHICH PROVIDED A FEASIBLE SOLUTION AND MARKED
C          ACCORDINGLY
C
      DO91J=1,JAT
      IX(2,J,LT)=0
      IXX( J,LT)=IP(2,IR)+IXX( J,1)
C
C     IF C(J) IS GREATER THAN THE MAXIMUM COMPUTED COST
C          AFTER WHICH ALL COLUMNS OF FRACTIONAL PARTS WILL
C          BE DUPLICATES AND BOTH MODES OF COLUMN ELIMINATION
C          HAVE BEEN UTILIZED, PROGRAM STOPS.

      IF(IXX(J,LT).GE.ISUM .AND.IOT.NE.1)GOTO171
      IF(IXX(J,LT).GE.ISUM)GOTO172
C
      IF(IALT.NE.1.AND.IXX(J,LT).EQ.ICJAY)IX(1,J,LT)=0
      IF(IXX(J,LT).GT.ICJAY.AND.LOKP.NE.1)IX(1,J,LT)=0
   91 IF(IXX(J,LT).GT.NCJAY)IX(1,J,LT)=0
C
C     NON-BASIC VARIABLE VALUES CARRIED FORWARD
C
      DO10I=1,JA
      IF(I.EQ.IR)GOTO10
      IX(IA,I,LT)=IP(IA,I)
   10 CONTINUE
      DO 90 I=3,IAT
      DO 90 J=1,JAT
      IF(IX(1,J,LT).EQ.0)GOTO840
      IF(J.EQ.IR)GOTO85
C
C     X(J) GREATER THAN NB(J)? IF SO MARK ENTIRE COLUMN
C
      IF(IX(IA,J ,LT).GE.NB(J ))GOTO81
      GOTO82
C
C     X(R)GREATER THAN NB(R)? IF SO MARK ENTIRE COLUMN
C
   85 IXA=IX(IA,J,LT)+1
      IF(IXA.GE.NB(J))GOTO81
      GOTO82
C
C     IF C(J) GREATER THAN CURRENT ICJAY, SET ENTIRE COLUMN
C          TO ZERO RATHER THAN COMPUTE
C
   81 IX(I,J ,LT)=0
      IX(1,J,LT)=0
      GOTO90
C
C     REMAINING FRACTIONAL PARTS OF COLUMNS COMPUTED
C
   82 ITP=IP(I,IR)+IX(I,J,1)
   84 ALK=LK
      AIM=ITP
      BIM=AIM/ALK
      IF(BIM.LT.1)IX(I,J,LT)=ITP
      IF(BIM.EQ.1)IX(I,J,LT)=0
      IF(BIM.GT.1)IX(I,J,LT)=ITP-LK
      GOTO90
  840 IX(I,J,LT)=0
   90 CONTINUE
C
C     LAST CCLUMN (I.E., ALPHA VALUES) CARRIED FORWARD
C
      DC99I=3,IA
```

```
   99 IX(I,JA,LT)=IX(I,JA,1)
      IXX(JA,LT)=IXX(JA,1)
C
C
C     NEW SECTION  COLUMNS CHECKED FOR DUPLICATES
C
C
      IF(LAP.NE.1)KAT=LT
      DO2J=1,JAT
      DO2I=1,KAT
      IF(I.EQ.LT)GOTO2
      DO2K=1,JAT
      IF(IXX(J,LT).NE.IXX(K,I))GOTO2
      DO22L=3,IAT
      RLK=IX(L,J,LT)-IX(L,K,I)
      IF(ABS(RLK).LE.YLK)GOTO22
      GOTO2
   22 CONTINUE
      IF(IXX(J,LT).EQ.ICJAY.AND.IALT.EQ.1)GOTO205
C
C     DUPLICATE COLUMNS CHECKED TO DETERMINE WHETHER OR NOT
C          SUBSEQUENT SECTIONS FORMED WILL HAVE IDENTICAL
C          NON-BASIC VARIABLE VALUES IF THEY ARE RETAINED
C          (I.E., UNMARKED AND USED FOR NEW SECTIONS) ***
C
C     THIS STEP BYPASSED ON FIRST TRY FOR SOLUTION AND USED
C          ON SECOMD TRY IF ITERATIONS HAD EXCEEDED 2000
C          OR REQUIRED SECTION SPACE EXCEEDED 450.
C
      IF(IOT.NE.1)GOTO218
  205 DO 216 KD=1,JAT
      IF(KD.EQ.J.OR.KD.EQ.K)GOTO216
      IF(IX(IA,KD,I).NE.IX(IA,KD,LT))GOTO2
  216 CONTINUE
      IXJLT=IX(IA,J,LT)+1
      IXKI=IX(IA,K,I)+1
      IF(J.EQ.K)GOTO217
      IF(IXJLT.NE.IX(IA,J,I))GOTO2
      IF(IXKI.NE.IX(IA,K,LT))GOTO2
      GOTO218
  217 IF(IX(IA,J,LT).NE.IX(IA,K, I))GOTO2
C
C     ALL DUPLICATE COLUMNS MARKED ON FIRST TRY
C     ONLY DUPLICATES WHICH MEET REQUIREMENT '***' ABOVE
C          ARE MARKED ON SECOND TRY
C
  218 IX(1,J,LT)=0
C
    2 CONTINUE
      IF(LAP.EQ.0)KAT=0
      IF(LAP.EQ.1)GOTO71
C
C     IF USED SPACE NOT AVAILABLE DETERMINE WHETHER OR NOT
C          NEW SECTION COLUMNS TOTALLY MARKED...IF SO,REUSE
C          SPACE
C
      DO911J=1,JAT
      IF(IX(1,J,LT).NE.0)GOTO71
  911 CONTINUE
      LT=LT-1
      GOTO511
   50 IPP=1
      GOTO71
  838 IOPP=1
      IOT=IOT+1
      IF(IOT.EQ.1.AND.IALT.EQ.1)GOTO881
      GOTO71
  881 WRITE(6,882)
  882 FORMAT(////' ALL EXISTING OPTIMAL SOLNS. SOLVED FOR ON
     1ADDITIONAL OPTIMA NOW SEARCHED FOR USING FULL DUPLICAT
      GOTO711
  171 ICT=IOT+1
```

```
      IOU=1
      CALLTIMEIT(-1,TIMEX)
      WRITE(6,542)TIMEX
  542 FORMAT(//' TIME SPENT ON ATTEMPT AT INTEGER PORTION OF
     1,-6PF15.6,' SECONDS.')
      IF(IOT.EQ.1.AND.IALT.NE.1)WRITE(6,543)
  543 FORMAT(//' FIRST ATTEMPT FAILED, SOLN. MAY HAVE BEEN B
     1BLEM NOW RERUN USING TOTAL DUPLICATE CHECK.')
      IF (IOT.EQ.1.AND.IALT.EQ.1.AND.MCR.LT.ICJAY.AND.
     1IXX(J,LT).LT.ISUM)WRITE(6,3)
      IF(IOT.EQ.1.AND.IXX(J,LT).GE.ISUM)WRITE(6,1711)
 1711 FORMAT(//' NO FEASIBLE SOLUTION AVAILABLE UNDER MODIFI
     1 COL. ELIMINATION METHOD..PROBLEM NOW RERUN USING TOTA
     1.')
    3 FORMAT(//' ALTERNATE OPTIMA STILL BEING SEARCHED FOR W
     1CE LIMITS REACHED..PROBLEM NOW RERUN USING TOTAL DUP.
      IF(IOT.EQ.2.AND.IALT.EQ.1.AND.MCR.LT.ICJAY)WRITE(6,4)
    4 FORMAT(//' ALTERNATE OPTIMA STILL BEING SEARCHED FOR W
     1CE LIMITS REACHED.')
  711 IF(IOT.EQ.1)WRITE(6,544)LKTT,LT
  544 FORMAT(/' ITERATIONS THUS FAR= ',I5,', SECTIONS REQUIR
      GOTO71
  172 JUMP=1
      ICT=IOT+1
   71 RETURN
      END
```

```
      SUBROUTINE FEAS(IR,M,N,IGNAT)
C
C
C     ROUTINE TO COMPUTE EACH POSSIBLE SOLUTION AND CHECK
C          FOR FEASIBILITY
C
C
      COMMON/ONE/XUB(24),X(24),IB(14),AX(14),XLB(24),H(14),
     1A(14,24),XI(14)
      COMMON/THREE/KBVAR(10),KB(10),MX(13),MB(23),NB(10),
     1AT(23,23),JX(13),AMX(13),MOX(22),IP(13,23),IXX(23,450)
     1,IIIP(22),NG(13),IX(13,23,450),IG(23,23)
      INTEGER*2 IX,IG
      COMMON/FOUR/LT,IA,JA,IT,NBV,LK,IPP,IAT,JAT,ICJAY,IOP,
     1IBJAY,YLK,LKTT,LAP,KAT,IBV,LIP,IOU,MCR,ISUM,ICOST
     1,JUMP,LOKP,LOKPT,NCJAY,MCJAY
      DO1I=2,IAT
      M=I-1
      AMT=0.0
      DO15J=1,JAT
      IF(J.NE.IR)GOTO6
      AP=IP(IA,J)+1
      GOTO15
    6 AP=IP(IA,J)
   15 AMT=AT(M,J)*AP+AMT
C
C     BASIC VARIABLES VALUES
C
      IF(I.NE.2)AMX(I)=AT(M,N)-AMT
      IF(I.NE.2)GOTO1
C
C     OBJECTIVE FUNCTION VALUE
C
      AMX(I)=AT(M,N)+AMT
    1 CONTINUE
      DO9I=2,IAT
      IF(AMX(I).GE.0.0)GOTO2
      B=AMX(I)-.5
      GOTO3
    2 B=AMX(I)+.5
    3 JX(I)=B
      IF(I.EQ.2)GOTO9
      ARK=JX(I)
      ALK=ABS(ARK)-ABS(AMX(I))
      ABT=ABS(ALK)
      IF(ABT.GE.0005)GOTO5
    9 CONTINUE
C
C     BASIC VARIABLE BOUNDS CHECKED
C
      DO4I=3,IAT
      IQ=I-2
      IF(JX(I).GT.KB(IQ))GOTO5
      IF(JX(I).LT.0)GOTO5
    4 CONTINUE
      GOTO16
    5 IOP=1
      GOTO17
   16 DO211I=1,IAT
  211 MX(I)=JX(I)
      LIP=LIP+1
C
C     FEASIBLE SOLUTION RECORDED
C
      CALLTIMEIT(-1,TIMEX)
      WRITE(6,542)TIMEX
  542 FORMAT(//' COMPUTING TIME SINCE LAST POSSIBLE INTEGER
     1-6PF15.6,' SECONDS.')
      WRITE(6,400)LIP,LKTT
  400 FORMAT(////1XI4,' TH POSSIBLE FEASIBLE SOLUTION RECORD
```

```
     1TERATIONS REQUIRED THUS FAR...',I10)
      WRITE(6,3004)
3004  FORMAT(//' NON BASIC VARIABLES INTEGER SOLUTION VALUES
      DO212I=1,JAT
      IF(I.EQ.IR)GOTO213
      MOX(I)=IX(IA,I,IT)
      WRITE(6,3002)MB(I)      ,IX(IA,I,IT)
3002  FORMAT(/' X( ',I2,' )=',I6)
      GOTO212
 213  IBP=IX(IA,IR,IT)+1
      MOX(I)=IBP
      WRITE(6,3003)MB(I)      ,IBP
3003  FORMAT(/' X( ',I2,' )=',I6)
 212  CONTINUE
      DO66I=1,IBV
      LZ=I+2
  66  WRITE(6,300)KBVAR(I),MX(LZ)
 300  FORMAT(//' INTEGER SOLN. VALUE FOR BASIC VARIABLE, X('
     1)
      IF(IGNAT.EQ.1)MX(2)=MX(2)*(-1)
      WRITE(6,301) MX(2)
 301  FORMAT(////' OPTIMAL OBJECTIVE FUNCTION INTEGER SOLUTI
      CALLTIMEIT(0,TIMEX)
  17  RETURN
      END
```

```
      SUBROUTINE TIMEIT(N,TIME)
C
C
C     N=0 STARTS CLOCK; N=-1 STOPS CLOCK
C
      IT=N+2
      GO TO (20,10),IT
   10 CALL TIMON(M)
      TIMEM=M
      RETURN
   20 CALL TIMOFF(M)
      TIME=M
      TIME=(TIMEM-TIME)*26.0
      RETURN
      END
C
C
C     ASSEMBLY LANGUAGE PROGRAM TO  CONDUCT TIMING ROUTINE
C
C
TIMEALL  CSECT
         ENTRY  TIMON,TIMOFF
TIMON    SAVE   (14,12)          ENTRY VIA -CALL TIMON(N)- 15.5
         USING  TIMON,12
         LR     12,15
         ST     13,TEMP1
         LA     13,SAVE1
         L      2,0(1,0)
         L      3,TOTIME
         ST     3,CLOCKR
         ST     3,0(2,0)
         STIMER TASK,TUINTVL=CLOCKR
         L      13,TEMP1
EXIT     RETURN (14,12),T,RC=0
TIMOFF   SAVE   (14,12)          ENTER VIA -CALL TIMOFF(N)-
         USING  TIMOFF,12
         LR     12,15
         ST     13,TEMP1
         LA     13,SAVE1
         L      2,0(1,0)
         TTIMER CANCEL
         ST     0,0(2,0)
         L      13,TEMP1
         RETURN (14,12),T,RC=0
         CNOP   0,4
TOTIME   DC     X'7FFFFFFF'
CLOCKR   DS     F
SAVE1    DS     18F
TEMP1    DS     F
         END
```

HALDI #1

```
4    9
  2.    3.    1.    2.    2.    1.    0.    0.    0.    3.
  2.    2.    1.    2.    0.    1.    0.    0.   -6.    0.
  1.    0.    0.    0.    0.    1.    0.    0.   -7.    0.
  1.    0.    0.    0.    0.
 18.   15.    0.    0.    0.
  0.    0.   -1.   -1.   -1.    0.    0.    0.    0.
  1.    1.   -1.   -1.   -1.   -1.   -1.   -1.   -1.
  0.    0.    0.    0.    0.    0.    0.    0.    0.
```

TIME TO COMPUTE CONTINUOUS SOLUTION = 0.019968 SECONDS.

RESULT
X(1)= 0.5082
X(2)= 0.8197
X(3)= 3.0492
X(4)= 5.7377

Z= -8.78688
HAVE FRACTIONS

COMPUTING TIME SINCE LAST POSSIBLE INTEGER SOLUTION = 12.293632 SECONDS.

```
   1 TH POSSIBLE FEASIBLE SOLUTION RECORDED...TOTAL ITERATIONS REQUIRED THUS FAR...   197

NON BASIC VARIABLES INTEGER SOLUTION VALUES...
X(  5 )=      7
X(  6 )=      4
X(  7 )=      1
X(  8 )=      0
X(  9 )=      0

INTEGER SOLN. VALUE FOR BASIC VARIABLE, X( 2 )=      0

INTEGER SOLN. VALUE FOR BASIC VARIABLE, X( 4 )=      0

INTEGER SOLN. VALUE FOR BASIC VARIABLE, X( 3 )=      0

INTEGER SOLN. VALUE FOR BASIC VARIABLE, X( 1 )=      0


OPTIMAL OBJECTIVE FUNCTION INTEGER SOLUTION=                7
```

```
ALL EXISTING OPTIMAL SOLUTICNS SOLVED FOR ON *COURSE* RUN..ADDITIONAL OPTIMA NOW SEARCHED
     FOR USING *FINE* DUPLICATE COLUMN ELIMINATION.
ITERATIONS THUS FAR=   247, SECTICNS REQUIRED=   52

COMPUTATION TIME SINCE LAST POSSIBLE INTEGER SOLUTION =       3.840512 SECONDS.

ABOVE SOLUTION(S) REPRESENT UPPER/LOWER BOUNDS ON THE  OPTIMAL SOLUTION..PROBLEM NOW
     RERUN USING *FINE* DUPLICATE COLUMN ELIMINATION.
IF THIS METHOD FAILS OR IS INCOMPLETE DUE TO TIME/SPACE LIMITATIONS, THIS/THESE BOUNDS
     IS/ARE THE BEST APPROXIMATION TO A SOLUTICN WHICH THE PROGRAM CAN COMPUTE.
```

HALDI #1

2.	3.	1.	2.	2.	1.	0.	0.	0.	3.
2.	2.	1.	2.	0.	1.	0.	0.	-6.	0.
1.	0.	0.	0.	0.	1.	0.	0.	-7.	0.
1.	0.	0.	0.	0.	1.				
18.	15.	0.	0.	0.					
0.	0.	-1.	-1.	-1.	0.	0.	0.	0.	
1.	1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	

TIME TO COMPUTE CONTINUOUS SOLUTION = 0.019968 SECONDS.

RESULT
X(1)= 0.5082
X(2)= 0.8197
X(3)= 3.0492
X(4)= 5.7377

Z= -8.78688
HAVE FRACTIONS

TIME SPENT ON ATTEMPT AT INTEGER PORTION OF SOLUTION = 263.151616 SECONDS.

ALTERNATE OPTIMA STILL BEING SEARCHED FOR WHEN TIME/SPACE LIMITS REACHED.

SECTION #1

5	6	7	8	9	0
11	51	46	40	35	-1608
14	15	175	1	161	150
98	105	127	7	29	135
96	129	102	33	6	9
16	174	17	158	1	93
0	0	0	0	0	0

COMMON DENOMINATOR USED TO COMPUTE FRACTIONAL PARTS = 183

SECTIONS REQUIRED EXCEEDS 450. TOTAL ITERATIONS THUS FAR = 768

HAVE FINISHED

BIBLIOGRAPHY

1. Greenberg, H., "A Dynamic Programming Solution to Integer Linear Programs," Journal of Mathematical Analysis and Applications, vol. 26, no. 2, May, 1969.

2. Dantzig, G. B., Linear Programming and Extensions. Princeton University Press, New Jersey, 1963.

3. Shapiro, J.F., "Dynamic Programming Algorithms for the Integer Programming Problem - I; the Integer Programming Problem Viewed as a Knapsack Type Problem", Operations Research, 16: 103-21, 1968.

4. Haldi, J., "25 Integer Programming Test Problems", Working Paper No. 43, Stanford University, Palo Alto, California, Dec. 1964, pp. 1-6, 12-13.

5. Gass, S. I., Linear Programming. McGraw-Hill Book Company, New York, 1964.

6. Gomory, R. E., "On the Relation Between Integer and Non-Integer Solutions to Linear Programs", Proceedings National Academy of Science, 53: 260-65, 1965.

7. Hillier, F. S. and Lieberman, G. J., Introduction to Operations Research. Holden-Day, Inc., San Francisco, California, 1968.

8. Muth, J. F. and Thompson, Gerald L., Industrial Scheduling. Prentice-Hall, Inc., New Jersey, 1963.

9. Hadley, G., Non-Linear and Dynamic Programming. Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1964.

10. IBM, IBM System/360 Operating System, Supervisor and Data Management Services, IBM Corporation Programming Publications, San Jose, California, 1968.

11. IBM, IBM System/360 Operating System, Linkage Editor, IBM Corporation Programming Publications, San Jose, California, 1968.

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7.	Capt. J.C. Arick 14007 Cove Lane Apt. 202 Rockville, Maryland	1

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13. ABSTRACT

An algorithm for the solution of integer linear programming problems is presented and programmed in Fortran IV for use on digital computers. The program incorporates an optional feature which provides all existing alternative optimal solutions. Solutions, computation times, and iteration requirements for each of thirteen test problems are summarized and discussed.

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